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A COMPARISON OF FORMULAE FOR THE PRACTICAL CALCULATION OF EVAPOTRANSPIRATION DURING THE DAYTIME

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by N Thompson

1. Introduction

The Penman - Monteith equation

$$\lambda E = \frac{\Delta' (R_N - G) + \rho c_p \delta q / r_a}{\Delta' + c_p (1 + r_s / r_a)/\lambda} \qquad (1)$$

where

$\Delta' = \partial q(sat)/\partial T$

$q(sat)$ = saturation specific humidity

δq = specific humidity deficit

r_a = resistance to transfer of heat and water vapour from mean source height in canopy to height at which δq is measured.

provides in principle an exact means of calculating evapotranspiration on any time scale, provided its various components are properly specified. The provision of accurate values of the bulk surface resistance r_s is very difficult, but for the purposes of this paper it will be assumed that they are available. A method by which the "exact" calculation may be made for daytime conditions was described by Thompson (1979). He noted that the values of the usual meteorological variables may differ over the short grass of the met enclosure and over the surface under consideration, but minimised errors due to this by calculating the values of these variables at 20 m height over the short grass and then assuming these would be the same at this height over other types of surface. Allowances were made for non-neutral atmospheric stability. The available energy $(R_N - G)$ is not usually measured directly for routine calculations of evapotranspiration and is estimated using various empirical formula: in particular the net long-wave radiation is calculated as the difference between downward (sky) radiation and upward radiation from the surface, both derived empirically by using the screen temperature, and the assumption that the surface temperature would be the same. In non-advection conditions the surface will be warmer than the overlying air during the day, and the net radiation overestimated therefore, by around 20% over stressed short crops in light winds. A rigorous allowance was made for this in the exact calculations described by Thompson (1979), the results of which were compared with a "simple" application of the Penman-Monteith equation which assumed neutral stability, and that the meteorological variables had the same values at measured height over both instrument enclosure and the surfaces under consideration.

Various simplified methods of calculating evapotranspiration, additional to the one described in the last few lines above, have been proposed, and the remainder of this paper describes some of these, compares their results to those from "exact" calculations, and attempts to select the one which appears most suitable for general use.

2. Model 1: Penman - Monteith with neutral stability assumption

This uses equation (1) with the standard calculation of aerodynamic resistance up to a reference height of 1.2 m:

$$r_a = \frac{6.25}{U_{10}} \ln\left(\frac{10}{z_o}\right) \ln\left(\frac{1.2}{z_o/5}\right) \qquad (2)$$

where

U_{10} = 10 m wind speed

z_o = roughness length for momentum transfer to the underlying surface

Equation (2) makes the usual assumption that the roughness length for heat and water vapour transfer is roughly $0.2\,z_o$. It is seen that the equation ignores the zero plane displacement height which strictly it should include. This implies that the standard (screen) measured temperature and specific humidity apply at a height of 1.2 m above the zero plane of the cropped surface, and the wind at a height of 10 m above the zero plane.

The assumption of neutral stability leads to near-equality of surface and screen temperature, and so equation (2) provides no direct means of correcting estimated available energy for elevated surface temperature.

3. Model 2: Penman - Monteith with neutral stability but including a direct correction for a raised surface temperature.

Using an idea of Monteith (1981), the Penman - Monteith equation may be re-derived to include specifically a correction to net radiation for an elevated surface temperature. As before, let R_N be the net radiation estimated empirically from hours of sunshine, screen temperature and screen specific humidity with the assumption of equality of surface and screen. The true available energy $R_{NC} - G$ is then

$$R_{NC} - G = R_N - G - \epsilon\sigma\left((273.1 + T_o)^4 - (273.1 + T_{SCR})^4\right) \qquad (3)$$

where

σ = 5.67×10^{-8} Wm^{-2} $(^oK)^{-4}$ (Stefan's Constant)

ϵ = emissivity ($\simeq 0.95$)

T_o = surface temperature (^oC)

To a first approximation

$$R_{NC} - G = R_N - G - 4\epsilon\sigma(273.1 + T_{SCR})^3(T_o - T_{SCR}) \qquad (4)$$

$$= R_N - G - B(T_o - T_{SCR})$$

The equations from which the modified combination equation is derived are then

$$H+\lambda E = R_N - G - B(T_o - T_{scR}) \quad (5)$$

$$r_a = \rho c_p (T_o - T_{scR})/H \quad (6)$$

$$r_s + r_a = \rho(q(sat)_o - q_{scR})/E \quad (7)$$

$$= \rho\left(q(sat)_o - q(sat)_{scR} + q(sat)_{scR} - q_{scR}\right)/E \quad (8)$$

$$= \rho(\Delta'(T_o - T_{scR}) + \delta q)/E \quad (9)$$

From (5) and (6)

$$\lambda E = R_N - G - (T_o - T_{scR})(B + \rho c_p/r_a) \quad (10)$$

Substitution for $T_o - T_{scR}$ from (9) into (10) leads to

$$\lambda E = R_N - G - \left(\frac{(r_s+r_a)}{\rho\Delta'}E - \frac{\delta q}{\Delta'}\right)\left(B + \rho c_p/r_a\right) \quad (11)$$

After re-arrangement

$$\lambda E = \frac{\Delta'(R_N - G) + \rho c_p \delta q\,(1 + B r_a/\rho c_p)/r_a}{\Delta' + c_p(1 + r_s/r_a)(1 + B r_a/\rho c_p)/\lambda} \quad (12)$$

4. Model 3: Penman-Monteith equation with Thom and Oliver "modified Penman" aerodynamic resistance

Thom and Oliver (1977) showed that Penman's (1948) combination equation used the equivalent of an aerodynamic resistance

$$r_a = 250/(1 + 0.54\,U_2) \quad (sm^{-1}) \quad (13)$$

where the reference level was at a height of 2 m, and the implied surface roughness length was 0.00137 m: U_2 is the wind speed at 2 m height. For a reference level of 1.2 m, and surface roughness length of z_o this equation becomes

$$r_a \simeq 220/(m(1 + 0.54\,U_2)) \quad (sm^{-1}) \quad (14)$$

with

$$m = \ln\left(\frac{1.2}{0.00137}\right)\ln\left(\frac{6}{0.00137}\right)\Big/\ln\left(\frac{1.2}{z_o}\right)\ln\left(\frac{6}{z_o}\right)$$

$$= 56.8/\left(\ln\left(\frac{1.2}{z_o}\right)\ln\left(\frac{6}{z_o}\right)\right) \qquad (15)$$

The particular significance of equation (14) is that it makes some attempt to correct r_a for the effects of an unstable surface layer. Thus, instead of r_a increasing rapidly to very large values as the wind speed decreases it converges to a value which should strictly reflect a state of free convection. Such a representation of the effects of instability is obviously fairly crude, since the approach to free convection depends not only on wind speed but also on the vertical flux of buoyancy.

The model uses this formulation for r_a in conjunction with the Penman-Monteith equation (equation (1)).

5. Model 4: Thom and Oliver's r_a + Penman-Monteith equation used with a direct correction for raised surface temperature

This uses equation (14) in conjunction with equation (12).

6. Model 5: The "exact" model

This is described by Thompson (1979). It reduces to equation (12), with B calculated at the mean of surface and screen temperatures, with r_a evaluated using standard flux-profile relations in order to allow for the effects of unstable density stratification in the surface layer of the atmosphere, and with a calculation scheme which provides a rough allowance for differing conditions over short grass and the surface in question as explained in § 1.

7. Results and discussion

The performance of models 1-4 may be judged by comparison with the results using the "exact" model 5. A large number of calculations is required for adequate comparisons, and it is not convenient to present these graphically. Tabulations are used instead (Table 1) where calculated differences between the results of each of the models 1-4 and values given by model 5 are expressed as percentages of the last. Results are given for the following values of variables:

$R_N - G$	30, 100, 200, 500 wm^{-2}
δq	0.001, 0.005
U_{10}	2, 5, 10 ms^{-1}
r_s	0, 30, 100, 500
z_o	0.01, 0.1, 1.0 m
T	5, 20 deg C

In many cases, more especially with stronger winds, low crop resistance and larger humidity deficit, the flux of buoyancy is towards the surface (stable conditions), circumstances which model 5 cannot deal with at present. Gaps appear in the Table in these cases.

A cursory glance at the Table shows that models 3 and 4 perform substantially less well on average than 1 and 2.

The choice between models 1 and 2 is less clearcut. Daytime average values of available energy are usually less than 200 Wm^{-2}, and the roughness lengths of most field crops lie in the range 0.01 - 0.1 m. Surface resistances are typically around 50 sm^{-1} in the main growing season, if soil moisture deficit does not limit transpiration. Average wind speeds are generally around, or a little less than, 5 ms^{-1}. In these circumstances the Table shows that model 2 gives results in marginally better agreement with the exact values than those of model 1. Its overall bias to a very small underestimation conveniently compensates to some extent for the neglect of the available energy used in photosynthesis by the transpiring crop.

It is recommended therefore that routine calculations of evapotranspiration should use equations (2) and (12).

The generally good performance of the simplest model (eq (1) and (2)) deserves some comment. This makes no specific correction for overestimation of available energy which follows from the assumption of equality of surface and screen temperature. In unstable conditions this would usually lead to overestimation of evapotranspiration if the aerodynamic resistance r_a had been calculated with proper corrections for the effects of diabatic conditions. However, eq (2) overestimates r_a in these conditions, and in most cases its use reduces the size of the calculated evapotranspiration therefore. Thus eq (1) and (2) provide some implicit correction for the effects of unstable density stratification.

Any comments or suggestions on this paper should be referred to the author

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References

Monteith, J L 1981 Evaporation and surface temperature. Q JR Met S, 107, 1-27.

Thom, A S and Oliver, H R 1977 On Penman's equation for estimating regional evaporation. Q J R Met S. 103, 345-357.

Thompson, N 1979 Towards a revised combination formula for practical calculations of potential and actual evaporation I. Daytime evaporation in non-advective conditions. Agricultural Memorandum No 859.

Table 1a $T = 5$, $z_o = 0.1$ m

δq	r_s	$R_N - G$	U = 2 E_5	E_1'	E_2'	E_3'	E_4'	U = 5 E_5	E_1'	E_2'	E_3'	E_4'	U = 10 E_5	E_1'	E_2'	E_3'	E_4'
0.001	0																
		100	78	-4	-7	49	50										
		200	129	-5	-11	27	25	168	-3	-4	23	23					
		500	288	-7	-15	7	4	315	-3	-5	11	10	390	-4	-5	8	7
	30																
		100	59	-1	-6	17	15	69	-1	-2	8	8	81	-3	-4	1	1
		200	95	2	-6	3	0	97	1	-2	2	0	98	0	-1	1	0
		500	205	3	-8	-10	-14	186	-1	-5	-10	-12	160	0	-2	-6	-7
	100	30	22	0	-2	16	16	26	-2	-3	4	3	28	-2	-2	0	0
		100	37	7	-1	-4	-6	35	3	0	-3	-4	33	2	1	0	-2
		200	58	13	1	-13	-16	49	3	-1	-10	-12	41	3	1	-3	-4
		500	120	18	3	-21	-25	95	1	-4	-20	-23	69	0	-2	-12	-14
	500	30	7	7	1	-6	-7	7	1	0	-1	-3	6	1	1	0	0
		100	11	20	8	-17	-20	9	4	1	-10	-11	7	5	4	-1	-2
		200	17	30	13	-23	-27	13	5	0	-17	-19	9	4	2	-5	-7
		500	35	40	18	-29	-33	25	3	-4	-26	-29	16	0	-3	-15	-17
0.005	0	500	427	-13	-16	35	36										
	30	200	191	-5	-6	36	36										
		500	319	-8	-12	8	7	350	-2	-3	8	7	397	-2	-2	3	2
	100	100	98	-2	-3	21	21										
		200	125	-2	-5	8	7	134	0	-1	4	3	142	-1	-1	1	1
		500	191	4	-4	-7	-9	179	0	-3	-5	-6	167	0	-1	-2	-3
	500	30	27	0	-1	6	5	29	0	-1	1	1	29	0	0	1	0
		100	32	4	0	-2	-3	31	1	0	-1	-1	31	1	0	0	0
		200	39	10	3	-8	-9	35	2	0	-4	-5	33	1	1	-1	-2
		500	57	22	9	-16	-19	47	1	-2	-13	-14	39	0	-1	-6	-6

Notes E_5 is the "exact"value of evapotranspiration (Wm^{-2}) calculated using model 5, § 6). $E_1' — E_4'$ are the differences between evapotranspiration calculated respectively by using models 1-4, and "exact" evapotranspiration (model 5), expressed as percentages of E_5.

Table 1b $T = 20$, $z_o = 0.10$ m

δq	r_s	$R_N - G$	U = 2 E_5	E_1'	E_2'	E_3'	E_4'	U = 5 E_5	E_1'	E_2'	E_3'	E_4'	U = 10 E_5	E_1'	E_2'	E_3'	E_4'
0.001	0	100	85	-1	-4	28	29										
		200	154	0	-5	16	15	181	-2	-3	13	13					
		500	363	-1	-7	6	4	384	0	-2	7	6	435	-3	-3	4	3
	30	100	71	3	-3	9	8	77	0	-1	5	4	84	-2	-2	1	1
		200	126	5	-3	0	-2	124	3	-1	-1	-3	118	2	1	0	-1
		500	292	6	-3	-6	-10	269	2	-2	-8	-10	231	2	0	-6	-7
	100	30	23	2	-1	11	10	26	-1	-2	3	3	28	-1	-2	0	0
		100	50	10	1	-8	-11	45	5	1	-5	-7	39	5	3	-1	-2
		200	87	15	3	-13	-17	72	5	0	-12	-15	56	6	3	-5	-6
		500	197	19	5	-17	-22	158	4	-2	-19	-22	113	3	1	-13	-15
	500	30	9	11	2	-12	-14	8	4	1	-4	-5	7	4	3	0	0
		100	18	26	9	-23	-27	13	8	2	-14	-17	10	7	5	-3	-5
		200	31	36	15	-26	-30	21	8	1	-20	-24	14	8	4	-8	-11
		500	68	45	21	-27	-33	47	7	-2	-27	-31	29	2	-2	-18	-20
.005	0	500	449	-6	-9	22	23										
	30	200	193	-3	-4	26	26										
		500	370	-1	-6	5	4	389	0	-2	5	4	418	-1	-2	2	1
	100	100	99	-1	-2	18	18										
		200	140	2	-3	4	2	144	1	0	2	1	147	0	0	1	0
		500	253	9	0	-8	-11	229	2	-1	-8	-9	201	1	0	-4	-5
	500	30	28	0	-1	5	4	29	-1	-1	1	1	30	0	-1	0	0
		100	37	9	2	-6	-8	34	2	1	-3	-4	32	2	1	0	-1
		200	50	18	7	-13	-15	43	4	0	-8	-10	37	3	1	-2	-3
		500	88	32	14	-19	-24	68	5	-1	-17	-20	51	2	1	-9	-11

Table 1c $T = 5$ $z_o = 0.01$ m

δq	r_s	$R_N - G$	U = 2					U = 5					U = 10				
			E_5	E_1'	E_2'	E_3'	E_4'	E_5	E_1'	E_2'	E_3'	E_4'	E_5	E_1'	E_2'	E_3'	E_4'
0.001	0	30	27	-3	-6	46	49										
		100	29	0	-13	22	17	76	0	-3	16	15	104	1	1	12	12
		200	108	0	-17	12	4	125	0	-6	10	6	152	0	-1	8	7
		500	268	-6	-25	-1	-11	280	-4	-12	0	-5	302	-2	-5	2	-1
	30	30	23	-1	-6	33	33										
		100	51	5	-11	14	7	58	2	-3	8	5	66	1	0	5	3
		200	92	5	-14	6	-4	95	3	-5	3	-2	97	1	-2	1	-1
		500	228	-1	-22	-7	-17	214	-2	-12	-7	-13	195	-3	-7	-6	-10
	100	30	18	3	-5	18	15	22	2	0	8	7	25	0	0	3	3
		100	38	13	-7	5	-4	37	6	-2	1	-3	36	2	-1	0	-2
		200	68	15	-9	-1	-13	61	6	-4	-4	-10	52	2	-1	-2	-7
		500	169	8	-18	-13	-25	139	0	-12	-14	-21	107	-4	-10	-12	-15
	500	30	8	16	-1	0	-5	7	4	0	-1	-4	7	1	0	0	-1
		100	15	35	3	-6	-16	12	10	-1	-6	-12	10	3	-1	-3	-7
		200	27	39	2	-11	-23	20	11	-3	-10	-18	14	2	-4	-7	-11
		500	68	28	-9	-23	-35	46	2	-13	-22	-30	30	-6	-13	-17	-22
0.005	0	200	171	-10	-16	27	28										
		500	337	-12	-24	7	3	401	-5	-8	10	9	524	0	0	11	11
	30	100	101	-6	-8	35	38										
		200	149	-8	-15	18	16	188	-1	-2	13	13					
		500	288	-7	-21	1	-5	308	-3	-8	2	0	337	-1	-2	3	1
	100	100	80	-4	-9	18	17	98	0	-1	9	9					
		200	112	-1	-12	7	3	122	0	-3	4	2	131	1	-1	2	1
		500	214	1	-17	-6	-14	199	-1	-8	-5	-10	185	-1	-4	-3	-6
	500	30	25	0	-2	9	8	27	0	0	3	3	29	0	0	1	1
		100	33	9	-3	1	-3	32	3	-1	0	-2	32	1	0	0	-1
		200	45	18	-2	-4	-10	40	4	-2	-3	-7	36	1	-1	-2	-4
		500	86	20	-8	-16	-26	66	1	-9	-14	-19	52	-3	-7	-9	-12

Table 1a $T = 20$ $z_o = 0.01$ m

δq	r_s	$R_N - G$	U = 2 E_5	E_1'	E_2'	E_3'	E_4'	U = 5 E_5	E_1'	E_2'	E_3'	E_4'	U = 10 E_5	E_1'	E_2'	E_3'	E_4'
0.001	0	30	27	0	-4	28	31										
		100	72	5	-7	15	11	85	1	-1	10	9	102	1	1	8	8
		200	138	5	-9	10	4	151	2	-3	7	4	170	1	0	5	4
		500	346	2	-13	4	-3	357	1	-5	3	-1	375	1	-2	3	0
	30	30	25	2	-4	22	22	33	-1	0	11	13					
		100	65	8	-6	11	5	71	4	-1	6	3	75	2	0	3	2
		200	124	9	-8	7	-1	126	5	-2	3	-2	125	2	-1	1	-2
		500	309	6	-12	1	-8	299	3	-5	-1	-7	279	1	-4	-2	-6
	100	30	21	7	-3	13	10	23	3	0	6	4	25	1	0	3	2
		100	53	16	-4	6	-4	51	8	-1	1	-5	47	4	-1	0	-3
		200	100	18	-5	2	-9	91	9	-2	-1	-9	78	4	-2	-2	-7
		500	250	14	-9	-3	-15	218	6	-6	-6	-15	176	0	-6	-7	-13
	500	30	10	24	1	-1	-10	9	8	0	-2	-7	8	2	0	-1	-4
		100	25	40	4	-4	-18	19	15	0	-5	-14	15	5	-1	-4	-9
		200	47	43	4	-6	-21	35	16	-1	-8	-18	25	5	-4	-7	-13
		500	120	37	-2	-13	-28	86	9	-8	-15	-26	58	-2	-10	-14	-21
0.005	0	100	109	-6	-5	30	36										
		200	179	-4	-10	18	18										
		500	388	-3	-14	7	3	444	-1	-3	6	5	516	0	0	6	7
	30	100	100	-4	-5	23	27										
		200	163	-1	-9	13	11	191	-1	-1	9	9					
		500	349	1	-12	4	-2	364	1	-4	3	0	382	1	-1	2	1
	100	100	84	0	-6	13	12	99	0	0	7	7					
		200	133	5	-7	7	1	139	2	-2	3	1	143	1	0	2	1
		500	283	9	-10	-1	-10	264	4	-5	-3	-8	239	1	-3	-3	-6
	500	30	25	1	-2	8	7	27	1	0	0	3	29	0	0	1	1
		100	41	18	0	1	-6	38	6	0	-1	-4	35	2	-1	-1	-2
		200	63	28	1	-2	-13	53	9	-1	-4	-10	45	3	-2	-2	-6
		500	136	30	-3	-10	-23	104	7	-6	-12	-20	78	-1	-7	-9	-14

Table 1e $T = 5$ $z_o = 1.0$ m

S_q	r_s	R_N-G	U = 2 E_5	E_1'	E_2'	E_3'	E_4'	U = 5 E_5	E_1'	E_2'	E_3'	E_4'	U = 10 E_5	E_1'	E_2'	E_3'	E_4'
0.001	0	500	366	-1	-2	414	414										
	30	100	77	0	0	27	27										
		200	98	0	-1	2	2	99	0	-1	1	1	101	-1	-1	0	0
		500	162	1	-2	-33	-33	134	1	0	-21	-21	103	17	17	2	2
	100	100	33	1	0	-8	-9	31	4	0	-1	-1	29	6	6	4	4
		200	43	1	-1	-27	-27	34	5	4	-10	-10	29	16	15	7	7
		500	69	4	1	-51	-51	49	0	-1	-33	-33	32	25	24	0	0
	500	30	6	0	0	-5	-6	6	3	3	2	0	6	3	3	2	2
		100	8	0	-1	-23	-23	6	8	6	-5	0	6	12	12	8	8
		200	10	2	0	-37	-38	7	5	4	-15	-15	6	21	21	9	9
		500	16	7	4	-60	-61	11	-1	-2	-37	-38	7	28	28	0	-1
0.005	30	500	384	0	0	27	26										
	100	200	139	1	0	8	7	149	-2	-3	1	0	153	-3	-4	-2	-2
		500	168	0	0	-9	-9	158	0	0	-4	-4	151	3	3	1	1
	500	100	31	0	0	-2	-3	30	1	1	0	0	30	1	1	1	1
		200	33	1	0	-8	-8	31	2	1	-2	-3	30	2	2	1	1
		500	39	3	1	-21	-21	34	1	0	-10	-10	31	6	5	0	0

Table 1f $T = 20$ $z_o = 1.0$ m

S_q	r_s	R_N-G	U=2 E_5	E_1'	E_2'	E_3'	E_4'	U=5 E_5	E_1'	E_2'	E_3'	E_4'	U=10 E_5	E_1'	E_2'	E_3'	E_4'
0.001	0	500	415	0	-1	220	221										
	30	100	82	0	-1	19	18										
		200	121	1	-1	-14	-14	108	5	5	-4	-5	92	18	18	11	11
		500	237	2	-1	-48	-48	182	2	1	-36	-36	107	43	42	5	5
	100	100	41	-1	-1	-24	-24	32	12	11	-3	-3	27	23	22	14	14
		200	61	0	-1	-45	-45	41	10	9	-21	-21	26	46	45	21	21
		500	115	6	2	-66	-66	74	1	-1	-50	-50	35	54	53	0	0
	500	30	7	1	1	-13	-13	6	12	12	5	5	6	11	9	7	7
		100	11	1	-2	-40	-40	7	16	16	-9	-9	5	37	35	19	19
		200	16	3	-1	-56	-56	9	11	10	-28	-28	5	58	56	23	23
		500	29	10	5	-72	-72	17	-1	-2	-56	-56	7	58	56	-3	-4
0.005	30	500	409	0	0	19	21										
	100	200	146	1	0	3	3	150	-1	-1	0	0	152	-1	-2	-1	-1
		500	204	2	0	-23	-23	176	1	1	-12	-12	151	10	9	2	2
	500	100	33	1	0	-8	-8	31	3	3	-1	-1	29	4	4	3	2
		200	38	1	0	-19	-19	33	3	3	-6	-6	29	9	9	4	4
		500	51	5	2	-38	-38	40	0	-1	-21	-21	31	14	13	0	0